ATTRIBUTE CONTROL CHART

Dr. Raghu Nandan Sengupta
Professor
Department of Industrial and Management Engineering



All figures are taken from(unless otherwise mentioned):

Introduction to Statistical process Control

Douglas. C Montgomery

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Attribute Charts

- Consider a glass container for a liquid product
- We examine the container and classify it into one of the two categories called conforming or non-conforming, depending on whether the container meets the requirements on one or more quality characteristics
- This is an example of attributes data
- Attributes charts are generally not as informative as variables charts because there is typically more information in a numerical measurement
- Attribute charts are particularly useful in service industries and in nonmanufacturing quality-improvement efforts

Control Chart of fraction non-conforming (p chart)

- The fraction nonconforming is defined as the ratio of the number of nonconforming items in a population to the total number of items in that population
- Of multiple quality characteristics even if one is not met, the item is said to be nonconforming
- Sometimes fraction conforming is also studied and we get a control chart on process yield

Statistical Principal of fraction non-conforming

- Basic underlying principle – Binomial Distribution
- Let p be the probability that a unit will not conform to the specifications
  - All units produced are independent
- If a random sample of n units is drawn and D is the number of Defective units, then D will follow a Binomial Distribution

$$P\{D = x\} = \binom{n}{x} p^x (1-p)^{n-x} \qquad x = 0,\ 1,\ \ldots,\ n$$

- Sample fraction nonconforming

$$\hat{p} = \frac{D}{n}$$

Development of the p chart

- When p is known:

$$\text{UCL} = p + 3\sqrt{\frac{p(1-p)}{n}}$$

$$\text{Center line} = p$$

$$\text{LCL} = p - 3\sqrt{\frac{p(1-p)}{n}}$$

- When p is not given:
  - First we estima $\hat{p}_i = \frac{D_i}{n} \quad i = 1, 2, \ldots, m$ ple
  - Then we estim[illegible] p for all the samples

$$\bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{\sum_{i=1}^{m} \hat{p}_i}{m}$$

p Chart Calculations

- Final we calculate the limits

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\text{Center line} = \bar{p}$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

- Limits calculated from initial samples should be treated as trial control limits
- Usually some target value of p is given for the chart

Example

Frozen orange juice concentrate is packed in 6-oz cardboard cans. These cans are formed on a machine by spinning them from cardboard stock and attaching a metal bottom panel. By inspection of a can, we may determine whether, when filled, it could possibly leak either on the side seam or around the bottom joint. Such a nonconforming can has an improper seal on either the side seam or the bottom panel. Set up a control chart to improve the fraction of nonconforming cans produced by this machine.

Sample Number	Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$	Sample Number	Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$
1	12	0.24	17	10	0.20
2	15	0.30	18	5	0.10
3	8	0.16	19	13	0.26
4	10	0.20	20	11	0.22
5	4	0.08	21	20	0.40
6	7	0.14	22	18	0.36
7	16	0.32	23	24	0.48
8	9	0.18	24	15	0.30
9	14	0.28	25	9	0.18
10	10	0.20	26	12	0.24
11	5	0.10	27	7	0.14
12	6	0.12	28	13	0.26
13	17	0.34	29	9	0.18
14	12	0.24	30	6	0.12
15	22	0.44		347	$\bar{p} = 0.2313$
16	8	0.16			

Solution

- First we estimate p

$$\bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{347}{(30)(50)} = 0.2313$$

- Then we estimate the control limits

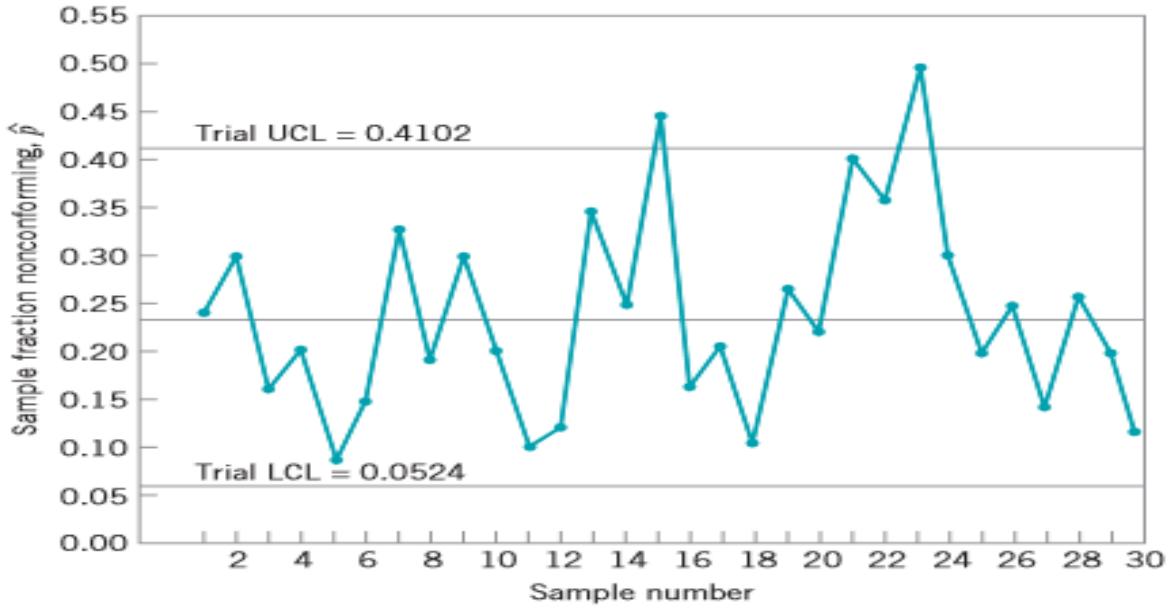
$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.2313 + 0.1789 = 0.4102$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.2313 - 0.1789 = 0.0524$$

- Based on the above calculation we plot the initial chart

Solution

- Points 15 and 23 are out of control



- It was found that point 15 and 23 were samples obtained from testing a new raw materials supplier and new operator (assignable causes)
- These points were removed and the limits were recalculated

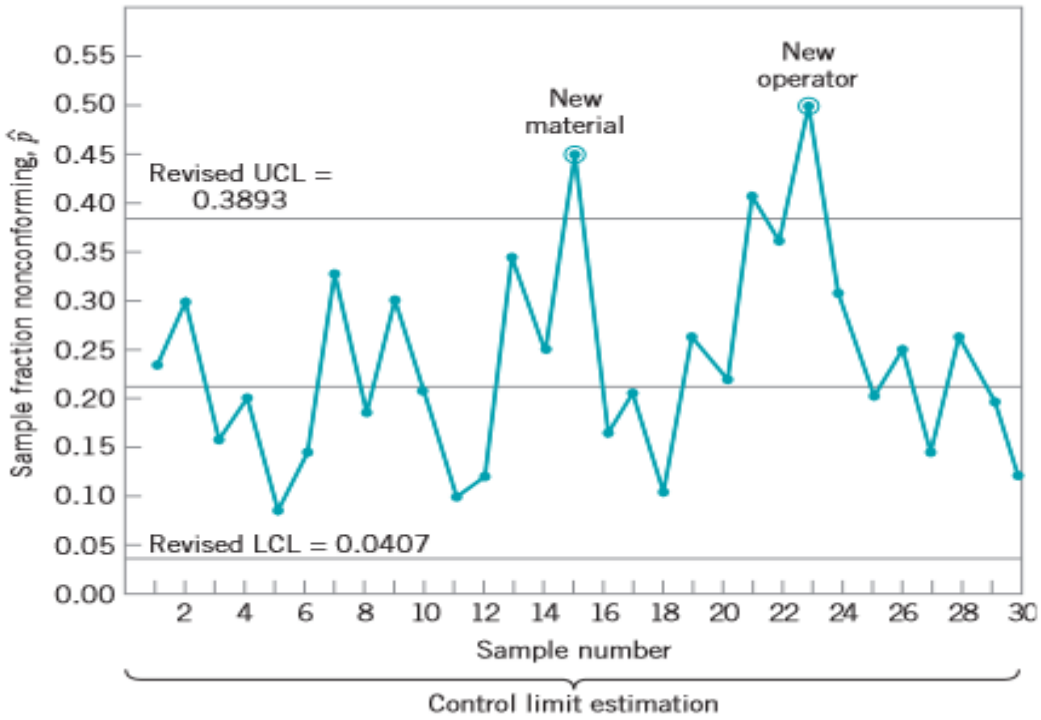
Solution

- The new points are calculated the chart is gain plotted

$$\bar{p} = \frac{301}{(28)(50)} = 0.2150$$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.2150 + 3\sqrt{\frac{0.2150(0.7850)}{50}} = 0.3893$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.2150 - 3\sqrt{\frac{0.2150(0.7850)}{50}} = 0.0407$$



Some considerations while designing the p chart (Calculating n)

- If p is very small, we should choose n sufficiently large so that we have a high probability of finding at least one nonconforming unit in the sample.
- Duncan (1986) has suggested that the sample size should be large enough that we have approximately a 50% chance of detecting a process shift of some specified amount

$$\delta = L\sqrt{\frac{p(1-p)}{n}}$$

  - Where L=number of si calculation and δ=magnitude of the shift
- If the in-control value of the fraction nonconforming is small, another useful criterion is to choose n large enough so that the control chart will have a positive lower control limit

$$n > \frac{(1-p)}{p}L^2$$

Example 7.4

7.4. A process that produces titanium forgings for automobile turbocharger wheels is to be controlled through use of a fraction nonconforming chart. Initially, one sample of size 150 is taken each day for twenty days, and the results shown in Table 7E.4 are observed.

(a) Establish a control chart to monitor future production.

(b) What is the smallest sample size that could be used for this process and still give a positive lower control limit on the chart?

TABLE 7E.4

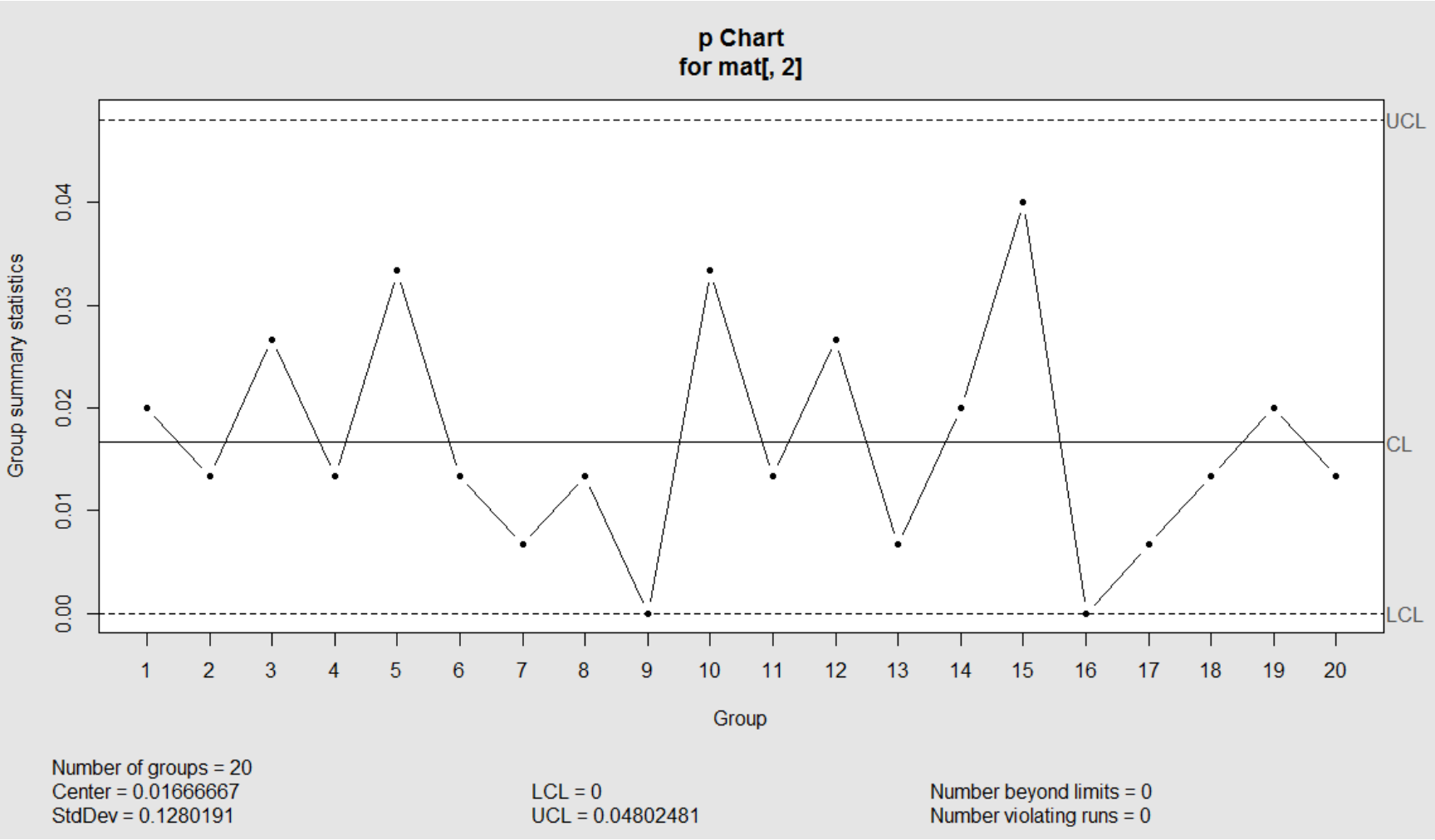
Nonconforming Unit Data for Exercise 7.4

Day	Nonconforming Units	Day	Nonconforming Units
1	3	11	2
2	2	12	4
3	4	13	1
4	2	14	3
5	5	15	6
6	2	16	0
7	1	17	1
8	2	18	2
9	0	19	3
10	5	20	2

R code

- mat<-matrix(nrow=20, ncol=2)
- mat[,1]<-seq(1:20)
- mat[,2]<-
  c(3,2,4,2,5,2,1,2,0,5,2,4,1,3,6,0,1,2,3,2)
- library(qcc)
- P<-qcc(mat[,2],type="p",sizes=150)

P chart output



Calculation of sample size

- Using formula

$$n > \frac{(1-p)}{p} L^2$$

- We use p$center to get estimate of p
  - L=3
- So n>((1-.0166)*9)/(.0166)
  - n>533.167 or n>534

The np control chart

- It is also possible to base a control chart on the number nonconforming rather than the fraction nonconforming
- Calculation of the limits

$$\text{UCL} = np + 3\sqrt{np(1-p)}$$
$$\text{Center line} = np$$
$$\text{LCL} = np - 3\sqrt{np(1-p)}$$

- If p is not available, p can be used
- It is easier to interpret

An example of np chart

Set up an *np* control chart for the orange juice concentrate can process in Example 7.1.

Data for Trial Control Limits, Example 7.1, Sample Size $n = 50$

Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$	Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$
1	12	0.24	17	10	0.20
2	15	0.30	18	5	0.10
3	8	0.16	19	13	0.26
4	10	0.20	20	11	0.22
5	4	0.08	21	20	0.40
6	7	0.14	22	18	0.36
7	16	0.32	23	24	0.48
8	9	0.18	24	15	0.30
9	14	0.28	25	9	0.18
10	10	0.20	26	12	0.24
11	5	0.10	27	7	0.14
12	6	0.12	28	13	0.26
13	17	0.34	29	9	0.18
14	12	0.24	30	6	0.12
15	22	0.44		347	$\bar{p} = 0.2313$
16	8	0.16			

Calculation of the control limits

$$\bar{p} = 0.2313 \qquad n = 50$$

$$\begin{aligned}
\text{UCL} &= n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} \\
&= 50(0.2313) + 3\sqrt{(50)(0.2313)(0.7687)} \\
&= 20.510 \\
\text{Center line} &= n\bar{p} = 50(0.2313) = 11.565 \\
\text{LCL} &= n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} \\
&= 50(0.2313) - 3\sqrt{(50)(0.2313)(0.7687)} \\
&= 2.620
\end{aligned}$$

Example 7.34

7.34. Consider the fraction nonconforming control chart in Exercise 7.5. Find the equivalent *np* chart.

■ TABLE 7E.5

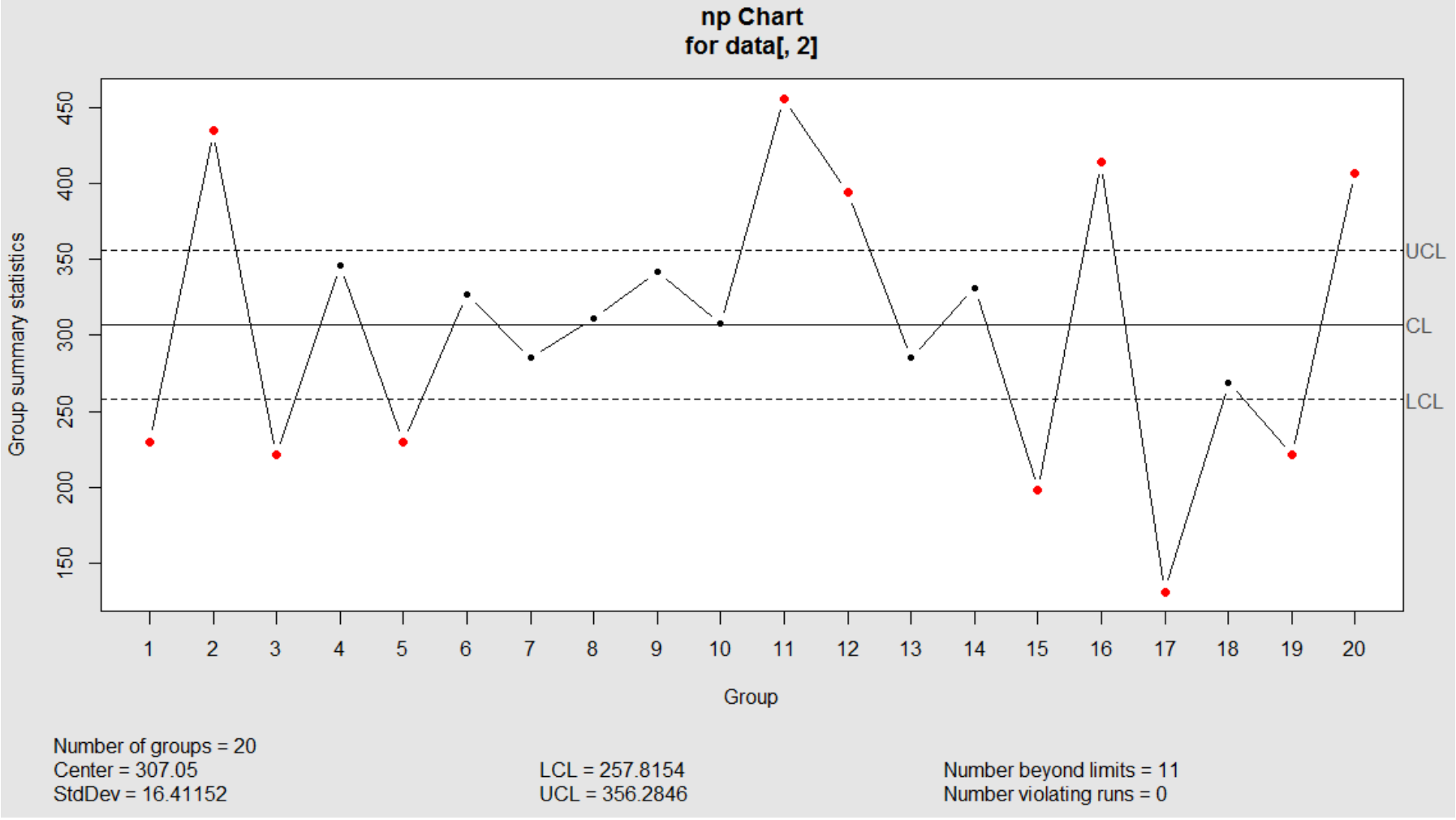
Inspection Data for Exercise 7.5

Lot Number	Number of Nonconforming Belts	Lot Number	Number of Nonconforming Belts
1	230	11	456
2	435	12	394
3	221	13	285
4	346	14	331
5	230	15	198
6	327	16	414
7	285	17	131
8	311	18	269
9	342	19	221
10	308	20	407

R code

- data<-matrix(nrow=20,ncol=2)
- data[,1]<-seq(1:20)
- data[,2]<-c(230,435,221,346,230,327,285,311,342,308,456,394,285,331,198,414,131,269,221,407)
- library(qcc)
- qcc(data[,2],sizes=2500,type="np")

Output


np Chart
for data[, 2]
Group summary statistics
150
200
250
300
350
400
450
UCL
CL
LCL
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
Group
Number of groups = 20
Center = 307.05
StdDev = 16.41152
LCL = 257.8154
UCL = 356.2846
Number beyond limits = 11
Number violating runs = 0

Variable sample size-1st method.

- Variable-Width Control Limits
- The most simple approach is to determine control limits for each individual sample that are based on the specific sample size
- That is, if the i^{th} sample is of size n_i, then the upper and lower control limits are

$$\bar{p} \pm 3\sqrt{\bar{p}(1-\bar{p})/n_i}.$$

An example

TABLE 7.4

Purchase Order Data for a Control Chart for Fraction Nonconforming with Variable Sample Size

Sample Number, i	Sample Size, n_i	Number of Nonconforming Units, D_i	Sample Fraction Nonconforming, $\hat{p}_i = D_i/n_i$	Standard Deviation $\hat{\sigma}_{\bar{p}} = \sqrt{\frac{(0.096)(0.904)}{n_i}}$	Control Limits LCL	Control Limits UCL
1	100	12	0.120	0.029	0.009	0.183
2	80	8	0.100	0.033	0	0.195
3	80	6	0.075	0.033	0	0.195
4	100	9	0.090	0.029	0.009	0.183
5	110	10	0.091	0.028	0.012	0.180
6	110	12	0.109	0.028	0.012	0.180
7	100	11	0.110	0.029	0.009	0.183
8	100	16	0.160	0.029	0.009	0.183
9	90	10	0.110	0.031	0.003	0.189
10	90	6	0.067	0.031	0.003	0.189
11	110	20	0.182	0.028	0.012	0.180
12	120	15	0.125	0.027	0.015	0.177
13	120	9	0.075	0.027	0.015	0.177
14	120	8	0.067	0.027	0.015	0.177
15	110	6	0.055	0.028	0.012	0.180
16	80	8	0.100	0.033	0	0.195
17	80	10	0.125	0.033	0	0.195
18	80	7	0.088	0.033	0	0.195
19	90	5	0.056	0.031	0.003	0.189
20	100	8	0.080	0.029	0.009	0.183
21	100	5	0.050	0.029	0.009	0.183
22	100	8	0.080	0.029	0.009	0.183
23	100	10	0.100	0.029	0.009	0.183
24	90	6	0.067	0.031	0.003	0.189
25	90	9	0.100	0.031	0.003	0.189
	2450	234	2.383			

Calculation of Control Limits

For the 25 samples, we calculate

$$\bar{p} = \frac{\sum_{i=1}^{25} D_i}{\sum_{i=1}^{25} n_i} = \frac{234}{2450} = 0.096$$

Consequently, the center line is at 0.096, and the control limits are

$$\text{UCL} = \bar{p} + 3\hat{\sigma}_{\hat{p}} = 0.096 + 3\sqrt{\frac{(0.096)(0.904)}{n_i}}$$

and

$$\text{LCL} = \bar{p} - 3\hat{\sigma}_{\hat{p}} = 0.096 - 3\sqrt{\frac{(0.096)(0.904)}{n_i}}$$

Method II

- Control Limits Based on an Average Sample Size
- This assumes that future sample sizes will not differ greatly from those previously observed

$$\bar{n} = \frac{\sum_{i=1}^{25} n_i}{25} = \frac{2450}{25} = 98$$

Therefore, the approximate control limits are

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}} = 0.096 + 3\sqrt{\frac{(0.096)(0.904)}{98}} = 0.185$$

and

$$\text{LCL} = \bar{p} - \sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}} = 0.096 - 3\sqrt{\frac{(0.096)(0.904)}{98}} = 0.007$$

Control chart for non-conformities(c chart)

- Each specific point at which a specification is not satisfied results in a defect or nonconformity.
- a nonconforming item will contain at least one nonconformity
- depending on their nature and severity, it is quite possible for a unit to contain several nonconformities and not be classified as nonconforming
  - suppose we are manufacturing personal computers. Each unit could have one or more very minor flaws in the cabinet finish, and since these flaws do not seriously affect the unit's functional operation, it could be classified as conforming
  - if there are too many of these flaws, the personal computer should be classified as nonconforming, since the flaws would be very noticeable to the customer and might affect the sale of the unit

Method III

- The Standardized Control Chart
- Such a control chart has the center line at zero, and upper and lower control limits of +3 and −3, respectively

$$Z_i = \frac{\hat{p}_i - p}{\sqrt{\frac{p(1-p)}{n_i}}}$$

  - Where p is process fraction nonconforming in the in-control state

Examples

- Method I

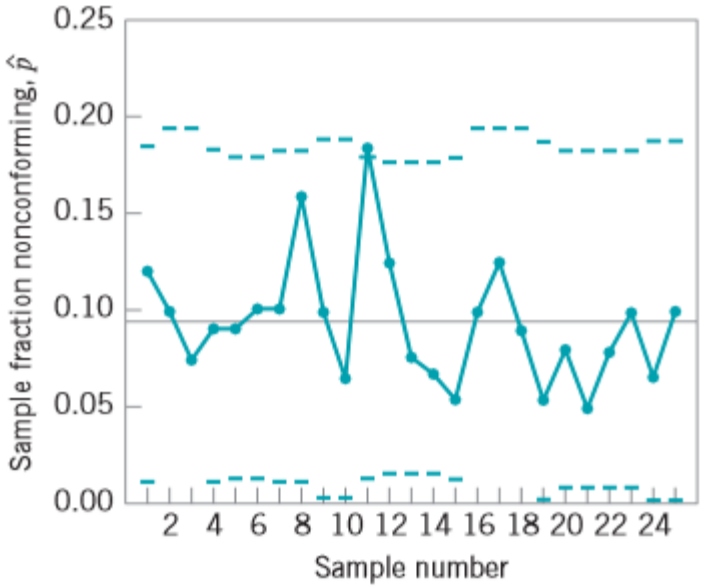


FIGURE 7.6 Control chart for fraction nonconforming with variable sample size.

- Method II

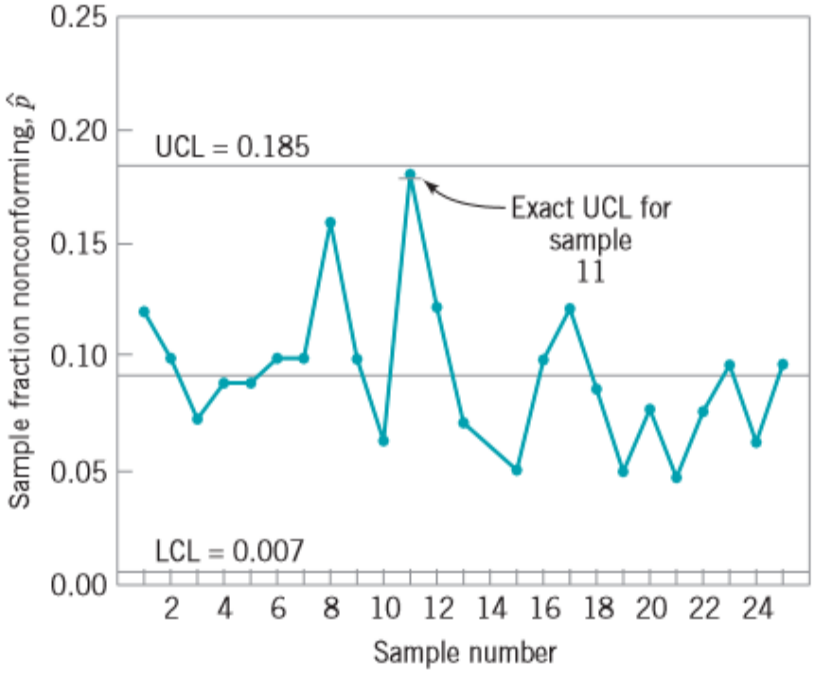


FIGURE 7.8 Control chart for fraction nonconforming based on average sample size.

Example Method III

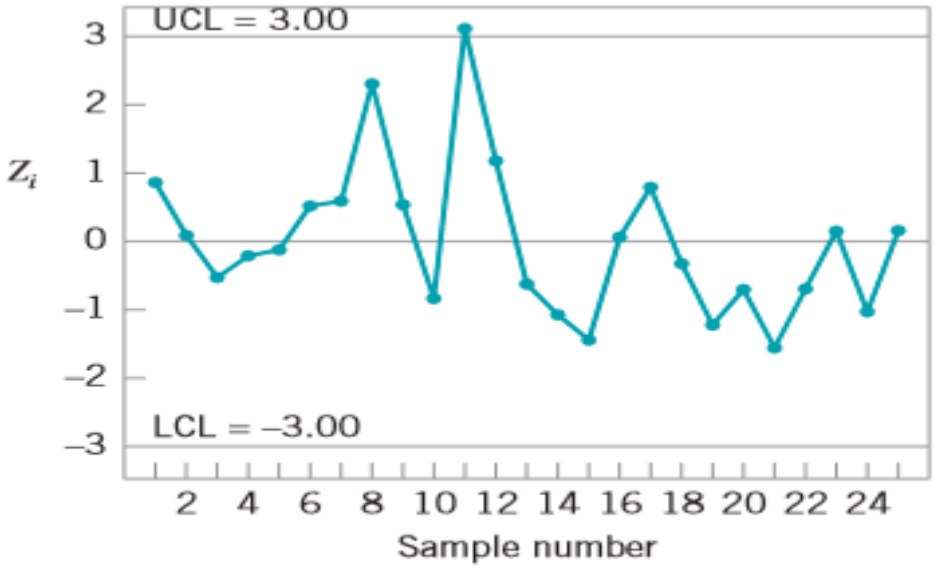


FIGURE 7.9 Standardized control chart for fraction nonconforming.

OC Function and Average Run Length

- The OC curve provides a measure of the sensitivity of the control chart—that is, its ability to detect a shift in the process fraction nonconforming from the nominal value p to some other value p.
- The probability of type II error for the fraction nonconforming control chart :

$$\beta = P\{\hat{p} < \text{UCL}|p\} - P\{\hat{p} \le \text{LCL}|p\}$$
$$= P\{D < n\text{UCL}|p\} - P\{D \le n\text{LCL}|p\}$$

- Since D is a binomial random variable with parameters n and p, the b-error defined in equation (7.15) can be obtained from the cumulative binomial distribution

OC Curve Calculation- An Example

Calculations[a] for Constructing the OC Curve for a Control Chart for Fraction Nonconforming with $n = 50$, LCL $= 0.0303$, and UCL $= 0.3697$

p	$P\{D \le 18 \mid p\}$	$P\{D \le 1 \mid p\}$	$\beta = P\{D \le 18 \mid p\} - P\{D \le 1 \mid p\}$
0.01	1.0000	0.9106	0.0894
0.03	1.0000	0.5553	0.4447
0.05	1.0000	0.2794	0.7206
0.10	1.0000	0.0338	0.9662
0.15	0.9999	0.0029	0.9970
0.20	0.9975	0.0002	0.9973
0.25	0.9713	0.0000	0.9713
0.30	0.8594	0.0000	0.8594
0.35	0.6216	0.0000	0.6216
0.40	0.3356	0.0000	0.3356
0.45	0.1273	0.0000	0.1273
0.50	0.0325	0.0000	0.0325
0.55	0.0053	0.0000	0.0053

[a]The probabilities in this table were found by evaluating the cumulative binomial distribution. For small p ($p < 0.1$, say) the Poisson approximation could be used, and for larger values of p the normal approximation could be used.

Example - continued

Table 7.6 illustrates the calculations required to generate the OC curve for a control chart for fraction nonconforming with parameters $n = 50$, LCL = 0.0303, and UCL = 0.3697. Using these parameters, equation (7.15) becomes

$$\beta = P\{D < (50)(0.3697)|p\} - P\{D \le (50)(0.0303)|p\}$$
$$= P\{D < 18.49|p\} - P\{D \le 1.52|p\}$$

However, since D must be an integer, we find that

$$\beta = P\{D \le 18|p\} - P\{D \le 1|p\}$$

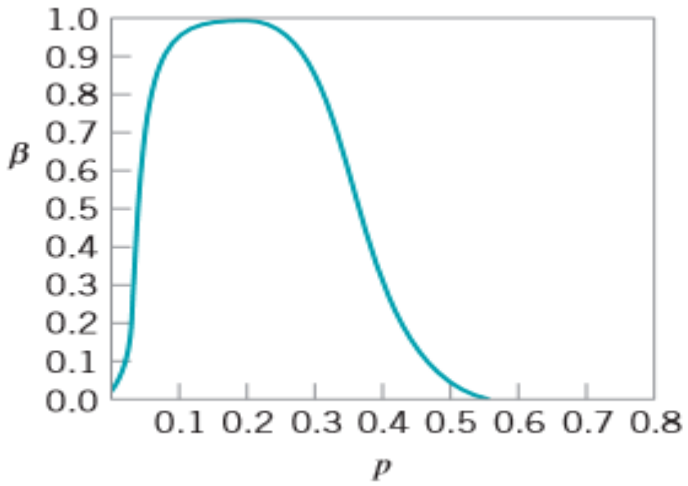


FIGURE 7.11 Operating-characteristic curve for the fraction nonconforming control chart with $\bar{p} = 0.20$, LCL = 0.0303, and UCL = 0.3697.

Average Run length when in control

- Consider the control chart for fraction nonconforming used in the OC curve calculations in Table 7.6. This chart has parameters n = 50, UCL = 0.3697, LCL = 0.0303, and the center line is $\bar{p}$. From Table 7.6 (or the OC curve in Fig. 7.11) we find that if the process is in control with p = $\bar{p}$, the probability of a point plotting in control is 0.9973. Thus, in this case $\alpha = 1 - \beta = 0.0027$, and the value of ARL_0 is

$$ARL_0 = \frac{1}{\alpha} = \frac{1}{0.0027} \simeq 370$$

ARL when process out of control

- Now suppose that the process shifts out of control to p = 0.3. Table 7.6 indicates that if p = 0.3, then β= 0.8594. Therefore, the value of ARL_1 is

$$ARL_1 = \frac{1}{1-\beta} = \frac{1}{1-0.8594} \simeq 7$$

Control Chart for Average Number of Non-conformities

- It is possible to develop control charts for either the total number of nonconformities in a unit or the average number of nonconformities per unit
- These control charts usually assume that the occurrence of nonconformities in samples of constant size is well modelled by the Poisson distribution.
  - This requires that the number of opportunities or potential locations for nonconformities be infinitely large and that the probability of occurrence of a nonconformity at any location be small and constant
  - The inspection unit must be the same for each sample

Calculation of limits

- Suppose that defects or nonconformities occur in this inspection unit according to the Poisson distribution

$$p(x) = \frac{e^{-c}c^{x}}{x!} \qquad x = 0,\ 1,\ 2,\ \ldots$$

- x is the number of nonconformities and c > 0 is the parameter of the Poisson distribution

$$\begin{aligned} \text{UCL} &= c + 3\sqrt{c} \\ \text{Center line} &= c \\ \text{LCL} &= c - 3\sqrt{c} \end{aligned}$$

- If calculated LCL<0, set LCL as 0

When c is not specified

- If no standard is given, then c may be estimated as the observed average number of nonconformities in a preliminary sample of inspection units - say, $\bar{c}$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}}$$
$$\text{Center line} = \bar{c}$$
$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}}$$

Example

Table 7.7 presents the number of nonconformities observed in 26 successive samples of 100 printed circuit boards. Note that, for reasons of convenience, the inspection unit is defined as 100 boards. Set up a *c* chart for these data.

TABLE 7.7

Data on the Number of Nonconformities in Samples of 100 Printed Circuit Boards

Sample Number	Number of Nonconformities	Sample Number	Number of Nonconformities
1	21	14	19
2	24	15	10
3	16	16	17
4	12	17	13
5	15	18	22
6	5	19	18
7	28	20	39
8	20	21	30
9	31	22	24
10	25	23	16
11	20	24	19
12	24	25	17
13	16	26	15

Calculation of the limits

Since the 26 samples contain 516 total nonconformities, we estimate c by

$$\bar{c} = \frac{516}{26} = 19.85$$

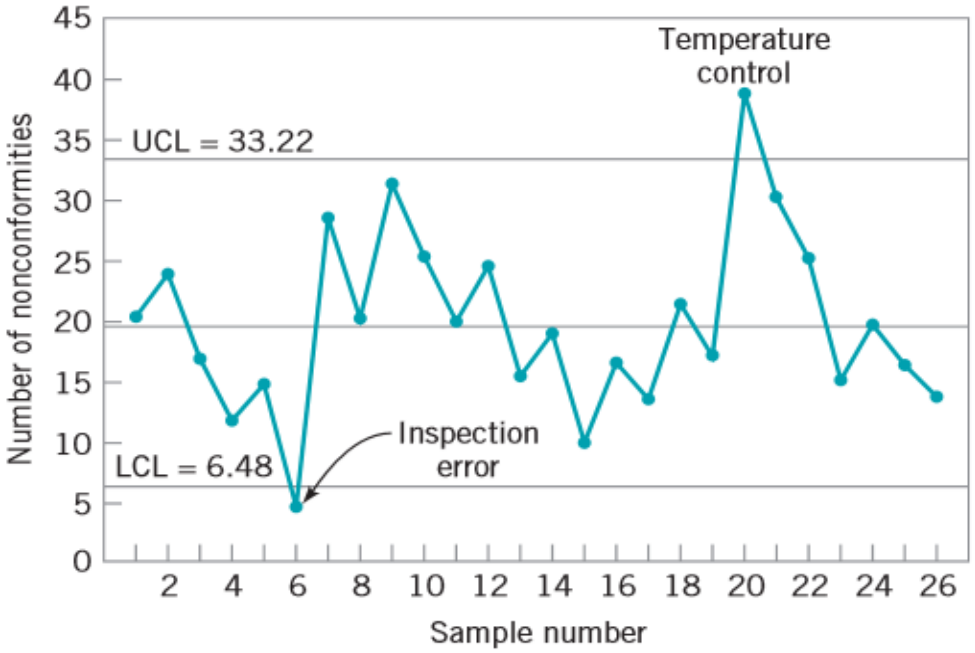
Therefore, the trial control limits are given by

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 19.85 + 3\sqrt{19.85} = 33.22$$

$$\text{Center line} = \bar{c} = 19.85$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 19.85 - 3\sqrt{19.85} = 6.48$$

C chart



- Two points plot outside the control limits, samples 6 and 20
- Sample 6-> new inspector
- Sample 20-> temperature control problem

Calculation of new control limits

- First new $\bar{c}$ is calculated

$$\bar{c} = \frac{472}{24} = 19.67$$

- The control limits are also recalculated

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 19.67 + 3\sqrt{19.67} = 32.97$$

$$\text{Center line} = \bar{c} = 19.67$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 19.67 - 3\sqrt{19.67} = 6.36$$

Example7.36

7.36. Surface defects have been counted on twenty-five rectangular steel plates, and the data are shown in Table 7E.10. Set up a control chart for nonconformities using these data. Does the process producing the plates appear to be in statistical control?

TABLE 7E.10

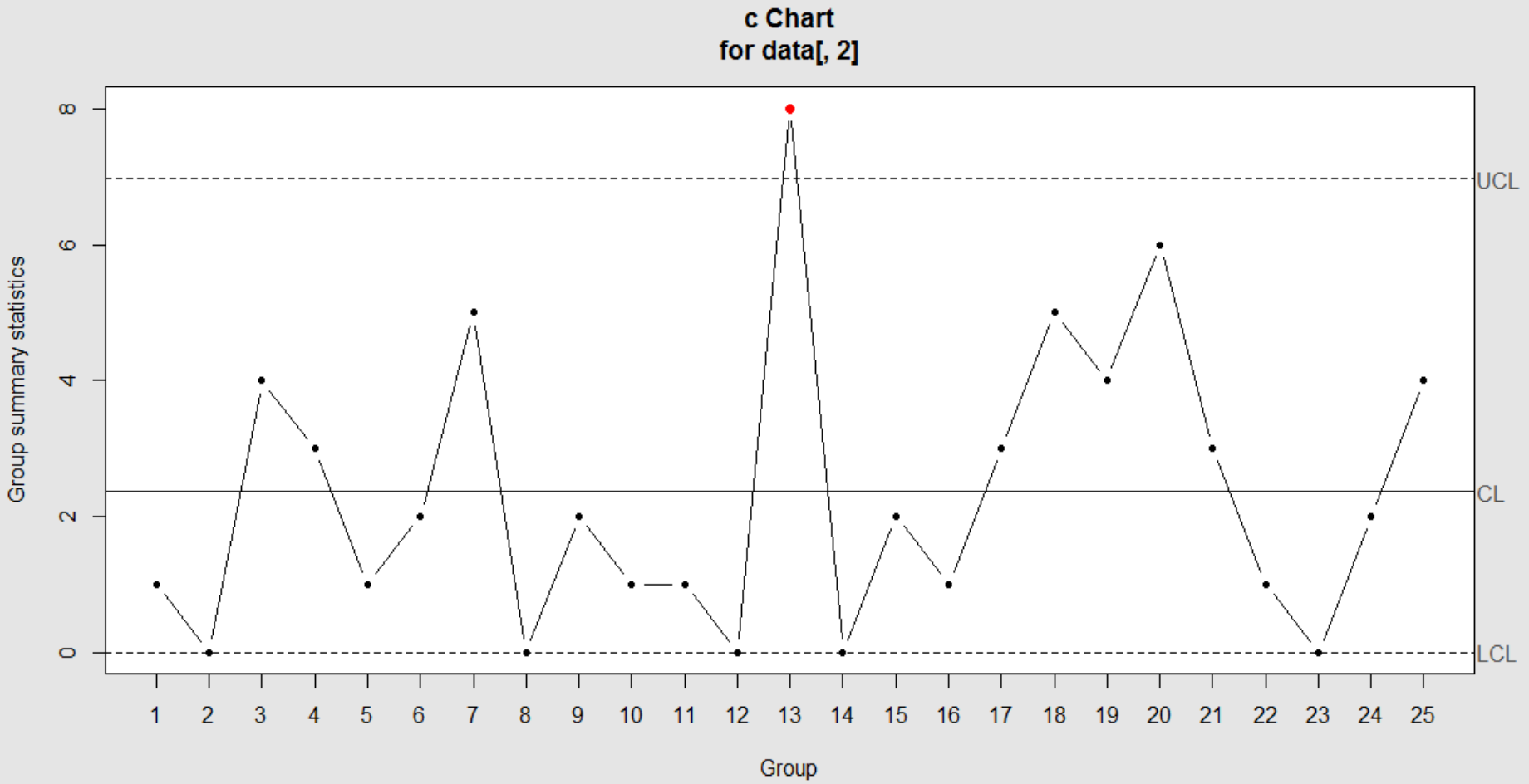
Data for Exercise 7.36

Plate Number	Number of Nonconformities	Plate Number	Number of Nonconformities
1	1	14	0
2	0	15	2
3	4	16	1
4	3	17	3
5	1	18	5
6	2	19	4
7	5	20	6
8	0	21	3
9	2	22	1
10	1	23	0
11	1	24	2
12	0	25	4
13	8		

R code

- data<-matrix(nrow=25,ncol=2)
- data[,1]<-seq(1:25)
- data[,2]<-
  c(1,0,4,3,1,2,5,0,2,1,1,0,8,0,2,1,3,5,4,6,3,1,0,2,
  4)
- library(qcc)
- qcc(data[,2],type="c",sizes=25)

Output


c Chart
for data[, 2]
Group summary statistics
0
2
4
6
8
UCL
CL
LCL
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
Group
Number of groups = 25
Center = 2.36
StdDev = 1.536229
LCL = 0
UCL = 6.968687
Number beyond limits = 1
Number violating runs = 0

The u chart(Average Number of non conformities)

- In the earlier case we assumed a 1 unit sample size
- However we can increase the sample size in which case when we create the chart, we have to consider average number of non conformities
- If we find x total nonconformities in a sample of n inspection units, then the average number of nonconformities per inspection unit is

$$u = \frac{x}{n}$$

Calculating control limits for the u chart

$$\text{UCL} = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$$

$$\text{Center line} = \bar{u}$$

$$\text{LCL} = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$$

- u represents the observed average number of nonconformities per unit

Example

A supply chain engineering group monitors shipments of materials through the company distribution network. Errors on either the delivered material or the accompanying documentation are tracked on a weekly basis. Fifty randomly selected shipments are examined and the errors recorded. Data for twenty weeks are shown in Table 7.10. Set up a u control chart to monitor this process.

Data on Number of Shipping Errors in a Supply Chain Network

Sample Number (week), i	Sample Size, n	Total Number of Errors (Nonconformities), x_i	Average Number of Errors (Nonconformities) per Unit, $u_i = x_i/n$
1	50	2	0.04
2	50	3	0.06
3	50	8	0.16
4	50	1	0.02
5	50	1	0.02
6	50	4	0.08
7	50	1	0.02
8	50	4	0.08
9	50	5	0.10
10	50	1	0.02
11	50	8	0.16
12	50	2	0.04
13	50	4	0.08
14	50	3	0.06
15	50	4	0.08
16	50	1	0.02
17	50	8	0.16
18	50	3	0.06
19	50	7	0.14
20	50	4	0.08
		74	1.48

Calculation of Control Limits

$$\bar{u} = \frac{\sum_{i=1}^{20} u_i}{20} = \frac{1.48}{20} = 0.0740$$

Therefore, the parameters of the control chart are

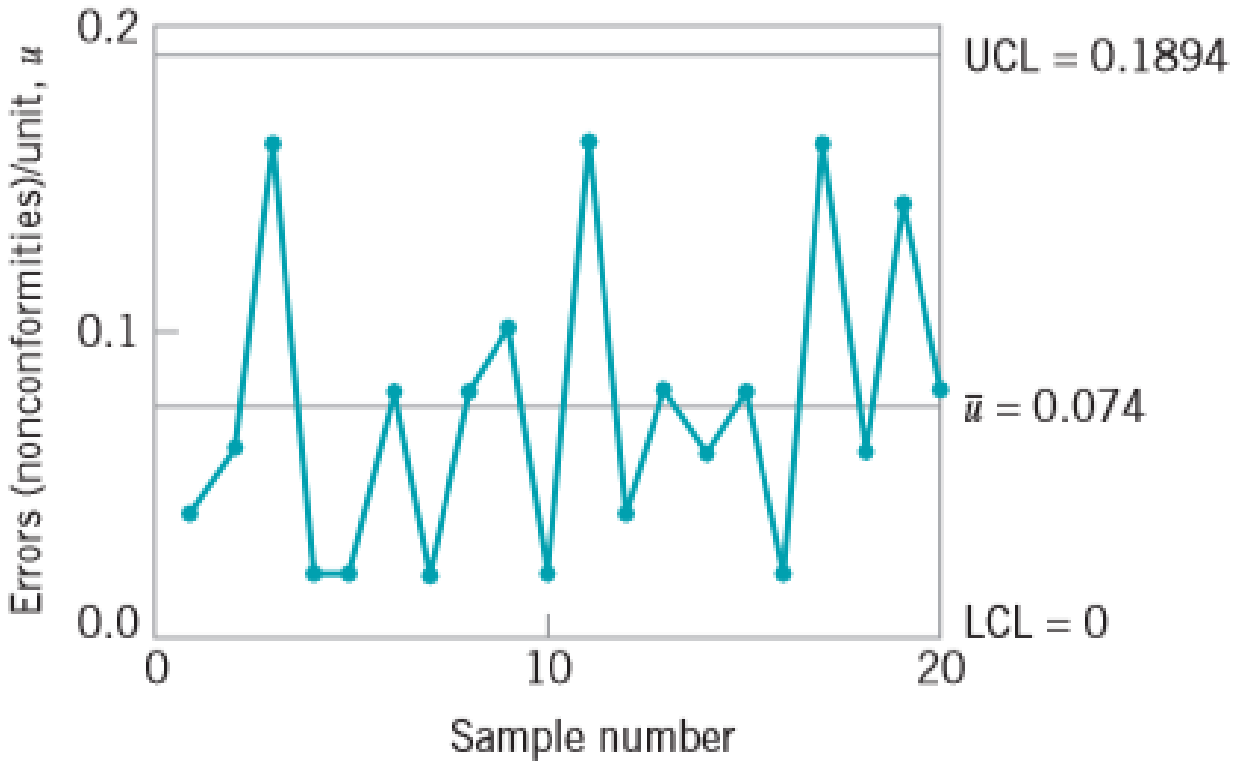
$$\text{UCL} = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 0.0740 + 3\sqrt{\frac{0.0740}{50}} = 0.1894$$

$$\text{Center line} = \bar{u} = 1.93$$

$$\text{LCL} = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 0.0740 - 3\sqrt{\frac{0.0740}{50}} = -0.0414$$

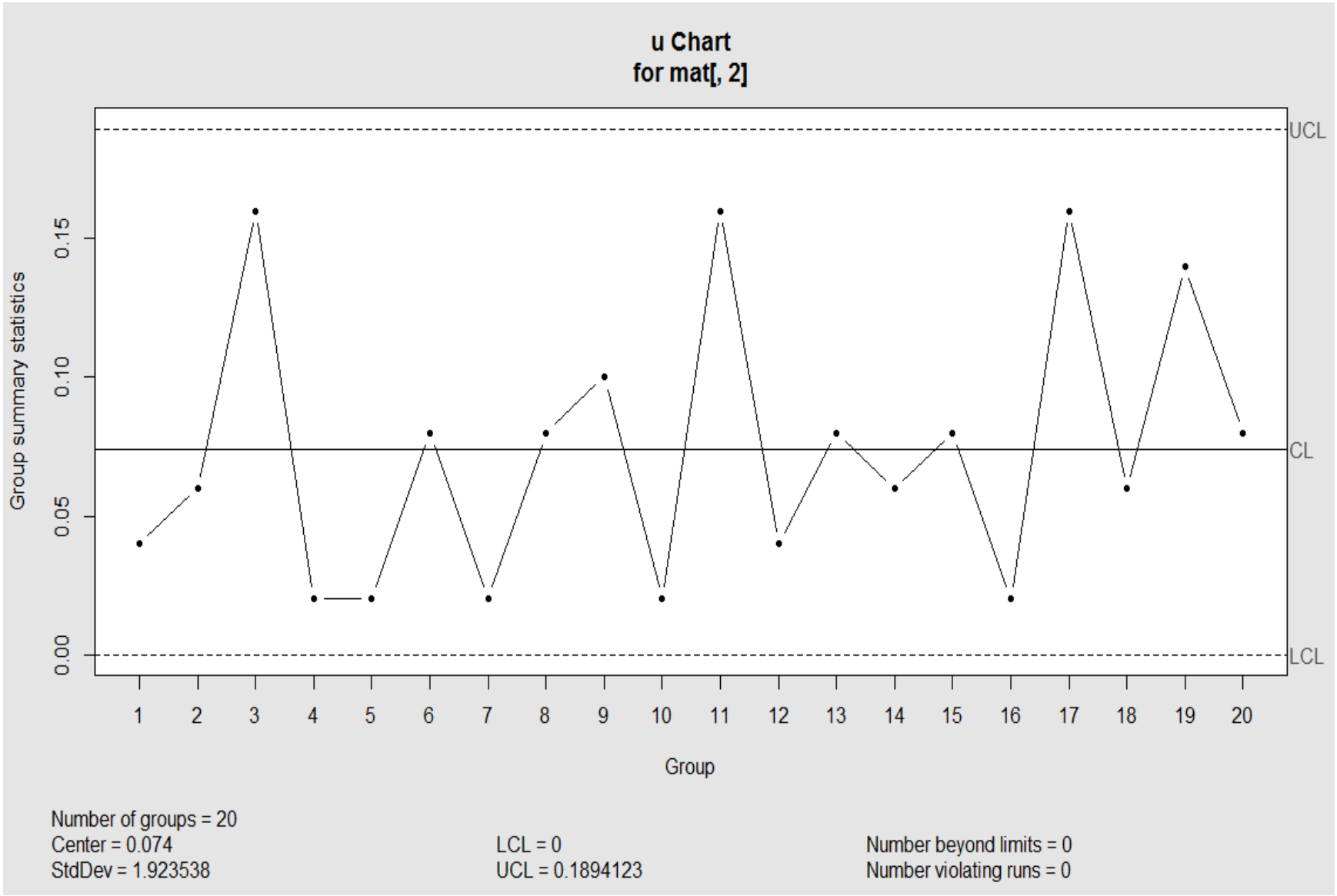
- Since LCL<0, for the chart we set LCL=0

Control Chart



R code

- mat<-matrix(nrow=20, ncol=2)
- mat[,1]<-seq(1:20)
- mat[,2]<-c(2,3,8,1,1,4,1,4,5,1,8,2,4,3,4,1,8,3,7,4)
- library(qcc)
- qcc(mat[,2],type="u",sizes=50)


u Chart
for mat[, 2]
Group summary statistics
0.00
0.05
0.10
0.15
UCL
CL
LCL
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
Group
Number of groups = 20
Center = 0.074
StdDev = 1.923538
LCL = 0
UCL = 0.1894123
Number beyond limits = 0
Number violating runs = 0

Alternative probability models for Count Data

- Most applications of the c chart assume that the Poisson distribution is the correct probability model underlying the process
- In the Poisson distribution, the mean and the variance are equal
  - This may not be always true
- Mixtures of various types of nonconformities can lead to situations in which the total number of nonconformities is not adequately modelled by the Poisson distribution
- Also when the count data have either too many or too few zeros

Continued

- The geometric distribution can also be useful as a model for count or “event” data
- Probability model for geometric distribution:

$$p(x) = p(1-p)^{x-a} \text{ for } x = a, a+1, a+2, \ldots$$

  - a is the known minimum possible number of events
- Suppose that the data from the process is available as a subgroup of size n, say $x_1, x_2, \ldots . X_n$
  - Independently and identically distributed observations

Continued

- The two statistics that can be used to form a control chart are the total number of events

$$T = x_1 + x_2 + \ldots + x_n$$

- and the average number of events

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

- Mean and variance for total number of events T are:

$$\mu_T = n\left(\frac{1-p}{p} + a\right) \qquad \sigma_T^2 = \frac{n(1-p)}{p^2}$$

Continued

- Mean and variance for average number of events:

$$\mu_{\bar{x}} = \frac{1-p}{p} + a \qquad \sigma^2_{\bar{x}} = \frac{1-p}{np^2}$$

- Kaminski et al. (1992) refer to the control chart for the total number of events as a “g chart” and the control chart for the average number of events as an “h chart.

Control limits for g and h charts

	Total number of events chart, g chart	Average number of events chart, h chart
Upper control limit (UCL)	$n\left(\frac{1-p}{p}+a\right)+L\sqrt{\frac{n(1-p)}{p^2}}$	$\frac{1-p}{p}+a+L\sqrt{\frac{1-p}{np^2}}$
Center line (CL)	$n\left(\frac{1-p}{p}+a\right)$	$\frac{1-p}{p}+a$
Lower control limit (LCL)	$n\left(\frac{1-p}{p}+a\right)-L\sqrt{\frac{n(1-p)}{p^2}}$	$\frac{1-p}{p}+a-L\sqrt{\frac{1-p}{np^2}}$

$$\hat{p}=\frac{1}{\bar{\bar{x}}-a+1}$$

Controls limits based on an estimate of p

- Suppose that there are m subgroups available, each of size n, and let the total number of events in each subgroup be t_1, t_2, . . . , t_m. The average number of events per subgroup is

$$\bar{t} = \frac{t_1 + t_2 + \cdots + t_m}{m}$$

	Total number of events chart, g chart	Average number of events chart, h chart
Upper control limit (UCL)	$\bar{t} + L\sqrt{n\left(\frac{\bar{t}}{n} - a\right)\left(\frac{\bar{t}}{n} - a + 1\right)}$	$\frac{\bar{t}}{n} + \frac{L}{\sqrt{n}}\sqrt{\left(\frac{\bar{t}}{n} - a\right)\left(\frac{\bar{t}}{n} - a + 1\right)}$
Center line (CL)	$\bar{t}$	$\frac{\bar{t}}{n}$
Lower control limit (LCL)	$\bar{t} - L\sqrt{n\left(\frac{\bar{t}}{n} - a\right)\left(\frac{\bar{t}}{n} - a + 1\right)}$	$\frac{\bar{t}}{n} - \frac{L}{\sqrt{n}}\sqrt{\left(\frac{\bar{t}}{n} - a\right)\left(\frac{\bar{t}}{n} - a + 1\right)}$

Procedure for variable sample size

- Control charts for nonconformities are occasionally formed using 100% inspection of the product.
- In this method the number of inspection units in a sample will usually not be constant
- If a control chart for nonconformities (c chart) is used in this situation, both the center line and the control limits will vary with the sample size. Such a control chart would be very difficult to interpret. The correct procedure is to use a control chart for nonconformities per unit (u chart).
- This chart will have a constant center line; however, the control limits will vary inversely with the square root of the sample size n.

Example

In a textile finishing plant, dyed cloth is inspected for the occurrence of defects per 50 square meters. The data on ten rolls of cloth are shown in Table 7.11. Use these data to set up a control chart for nonconformities per unit.

■ TABLE 7.11

Occurrence of Nonconformities in Dyed Cloth

Roll Number	Number of Square Meters	Total Number of Nonconformities	Number of Inspection Units in Roll, n	Number of Nonconformities per Inspection Unit
1	500	14	10.0	1.40
2	400	12	8.0	1.50
3	650	20	13.0	1.54
4	500	11	10.0	1.10
5	475	7	9.5	0.74
6	500	10	10.0	1.00
7	600	21	12.0	1.75
8	525	16	10.5	1.52
9	600	19	12.0	1.58
10	625	23	12.5	1.84
		153	107.50	

Calculations

$$\bar{u} = \frac{153}{107.5} = 1.42$$

TABLE 7.12

Calculation of Control Limits, Example 7.5

Roll Number, i	n_i	UCL $= \bar{u} + 3\sqrt{\bar{u}/n_i}$	LCL $= \bar{u} - 3\sqrt{\bar{u}/n_i}$
1	10.0	2.55	0.29
2	8.0	2.68	0.16
3	13.0	2.41	0.43
4	10.0	2.55	0.29
5	9.5	2.58	0.26
6	10.0	2.55	0.29
7	12.0	2.45	0.39
8	10.5	2.52	0.32
9	12.0	2.45	0.39
10	12.5	2.43	0.41

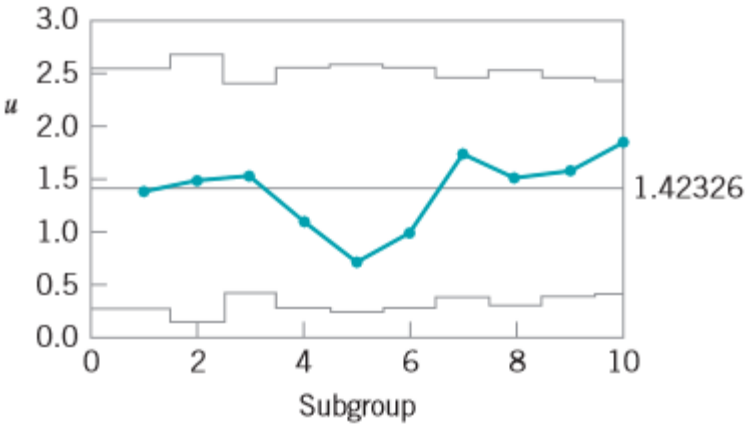


FIGURE 7.17 Computer-generated (Minitab) control chart for Example 7.5.

OC Function

- The operating-characteristic (OC) curves for both the c chart and the u chart can be obtained from the Poisson distribution. For the c chart, the OC curve plots the probability of type II error β against the true mean number of defects c. The expression for β is

$$\beta = P\{x < \text{UCL}|c\} - P\{x \leq \text{LCL}|c\}$$

An example

We will generate the OC curve for the c chart in Example 7.3. For this example, since the LCL = 6.48 and the UCL = 33.22, equation (7.26) becomes

$$\beta = P\{x < 33.22|c\} - P\{x \le 6.48|c\}$$

Since the number of nonconformities must be integer, this is equivalent to

$$\beta = P\{x \le 33|c\} - P\{x \le 6|c\}$$

These probabilities are evaluated in Table 7.13. The OC curve is shown in Fig. 7.19.

For the u chart, we may generate the OC curve from

$$\begin{aligned}\beta &= P\{x < \text{UCL}|u\} - P\{x \le \text{LCL}|u\} \\ &= P\{c < n\text{UCL}|u\} - P\{c \le n\text{LCL}|u\} \\ &= P\{n\text{LCL} < x \le n\text{UCL}|u\} \\ &= \sum_{x=\langle n\text{LCL}\rangle}^{[n\text{UCL}]} \frac{e^{-nu}(nu)^x}{x!}\end{aligned} \tag{7.27}$$

Example - Continued

TABLE 7.13

Calculation of the OC Curve for a c Chart with UCL = 33.22 and LCL = 6.48

c	$P\{x \le 33 \mid c\}$	$P\{x \le 6 \mid c\}$	$\beta = P\{x \le 33 \mid c\} - P\{x \le 6 \mid c\}$
1	1.000	0.999	0.001
3	1.000	0.966	0.034
5	1.000	0.762	0.238
7	1.000	0.450	0.550
10	1.000	0.130	0.870
15	0.999	0.008	0.991
20	0.997	0.000	0.997
25	0.950	0.000	0.950
30	0.744	0.000	0.744
33	0.546	0.000	0.546
35	0.410	0.000	0.410
40	0.151	0.000	0.151
45	0.038	0.000	0.038

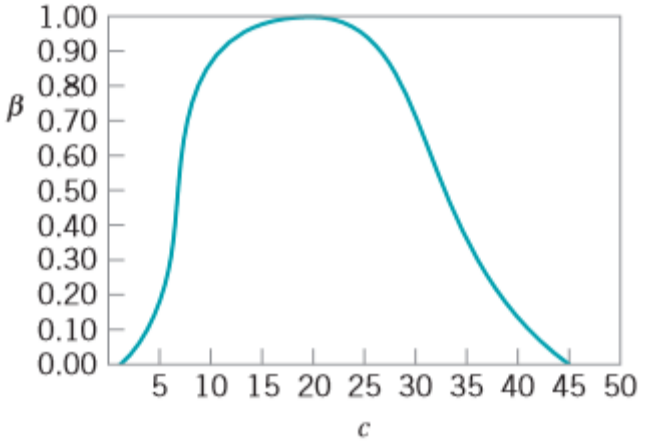


FIGURE 7.19 OC curve of a c chart with LCL = 6.48 and UCL = 33.22.

Low defects

- When defect levels in a process become very low—say, under 1000 occurrences per million—there will be very long periods of time between the occurrence of a nonconforming unit.
- In these situations, many samples will have zero defects, and a control chart with the statistic consistently plotting at zero will be relatively uninformative
- Conventional c and u charts become ineffective
- Create a time between occurrence control chart, which charts a new variable: the time between the successive occurrences of the count.

Creating time between occurrence control chart

- Suppose that defects or counts or “events” of interest occur according to a Poisson distribution. Then the probability distribution of the time between events is the exponential distribution
- the exponential distribution is highly skewed, and as a result, the corresponding control chart would be very asymmetric
- If y represents the original exponential random variable, the appropriate transformation is

$$x = y^{1/3.6} = y^{0.2777}$$

Choice between Attribute and Variable Control Chart

- Attributes control charts have the advantage that several quality characteristics can be considered jointly and the unit classified as nonconforming if it fails to meet the specification on any one characteristic.
- If the several quality characteristics are treated as variables, then each one must be measured, and either a separate and R chart must be maintained on each or some multivariate control technique that considers all the characteristics must simultaneously be employed.
- Expensive and time-consuming measurements may sometimes be avoided by attributes inspection

Choice between Attribute and Variable Control Chart

- Variables control charts, in contrast, provide much more useful information about process performance than does an attributes control chart. Specific information about the process mean and variability is obtained directly.
- In addition, when points plot out of control on variables control charts, usually much more information is provided relative to the potential cause of that out-of-control signal.
- For a process capability study, variables control charts are almost always preferable to attributes control charts.
  - Except studies relative to nonconformities produced by machines or operators in which there are a very limited number of sources of nonconformities
- $\bar{x}$ and R charts are leading indicators of trouble, whereas p charts (or c and u charts) will not react unless the process has already changed

Choosing chart type $\bar{x}$ or R chart

- A new process is coming on stream, or a new product is being manufactured by an existing process.
- The process has been in operation for some time, but it is chronically in trouble or unable to hold the specified tolerances.
- The process is in trouble, and the control chart can be useful for diagnostic purposes (troubleshooting).
- Destructive testing (or other expensive testing procedures) is required.
- It is desirable to reduce acceptance-sampling or other downstream testing to a minimum when the process can be operated in control.

Choosing chart type $\bar{x}$ or R chart

- Attributes control charts have been used, but the process is either out of control or in control but the yield is unacceptable.
- There are very tight specifications, overlapping assembly tolerances, or other difficult manufacturing problems.
- The operator must decide whether or not to adjust the process, or when a setup must be evaluated.
- A change in product specifications is desired.
- Process stability and capability must be continually demonstrated, such as in regulated industries.

Attributes Charts (p charts, c charts, and u charts).

- Operators control the assignable causes, and it is necessary to reduce process fallout.
- The process is a complex assembly operation and product quality is measured in terms of the occurrence of nonconformities, successful or unsuccessful product function, and so forth. (Examples include computers, office automation equipment, automobiles, and the major subsystems of these products.)
- Process control is necessary, but measurement data cannot be obtained.

Attributes Charts (p charts, c charts, and u charts).

- A historical summary of process performance is necessary. Attributes control charts, such as p charts, c charts, and u charts, are very effective for summarizing information about the process for management review.
- Remember that attributes charts are generally inferior to charts for variables. Always use and R or and s charts whenever possible.